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Left–Right Symmetry and Neutrino Masses in a Non–Perturbative Unification Framework

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Abstract

Within the non–perturbative unification framework pioneered by Maiani, Parisi and Petronzio, it is shown how the mass of the right–handed W boson can be determined just by knowing the values of $\sin^2\theta_w$ and $\alpha_3(\equiv g_3^2/4\pi)$ at the Fermi scale $\Delta_F \simeq 250$ GeV. Consequently, the knowledge of M_{W_R} helps to determine the light Majorana neutrino mass which is $m_\nu \simeq 0(1/M_{W_R})$. The best bounds we can obtain here are $M_{W_R} \gtrsim$ 8 TeV, $m_{\nu_e} \lesssim 5\times10^{-6}eV$, $m_{\nu_\mu} \lesssim 0.2eV$, $m_{\nu_\tau} \lesssim 65ev$.

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The mystery of parity violation in the electroweak interactions has sparked numerous investigations into its possible origin. In the standard model as well as in its simplest Grand Unified extension, i.e., $SU(5)$, parity violation is intrinsic and can only be understood if one knows the origin of the standard particle spectrum and its gauge interactions. On the other hand, it is quite possible that parity is spontaneously broken at some scale, Λ_R, above which it is restored. This is the example of left–right symmetric models[1] of the type $SU(2)_L \times SU(2)_R \times U(1)$. This idea is quite attractive and it would be very helpful if one could get a handle on the value of Λ_R. The $SU(2)_L$ breaking scale or the Fermi scale Λ_F is determined experimentally to be $\Lambda_F = (\sqrt{2}G_F)^{-1} \simeq 250$ GeV. No such determination is found for Λ_R except for the lower limits on the mass of the $SU(2)_R$ gauge bosons, namely $M_{W_R} > 300$ GeV or 1 TeV.

Is it somehow possible to relate the two scales Λ_F and Λ_R? This is the question we would like to address in this letter. It turns out that under a certain set of reasonable assumptions, such a relationship does exist. In fact, these assumptions allow us to calculate $\sin^2\theta_w(\Lambda_F)$ and $\alpha_3(\Lambda_F)$ and such computations, in turn, determine where Λ_R is with respect to Λ_F. It also turns out that this determination has deep implications on neutrino masses.

The concept employed in this note was advanced by Maiani, Parisi, and Petronzio[2] several years ago. It is the assumption that the "low" energy couplings of the standard model[3] are the near–infrared stable fixed points of a non-asymptotically free theory. It means two things: At $\Lambda_F, \alpha_3(\Lambda_F), \alpha_2(\Lambda_F)$, and $\alpha_y(\Lambda_F)$ (of $SU(3)_c \times SU(2)_L \times U(1)_y$) are small; above Λ_F, $SU(3)_c$ and $SU(2)_L$ are themselves non–asymptotically free ($U(1)_y$ already is) and all couplings grow large at some scale Λ_G at which point perturbation theory ceases to be valid (this is the famous Landau singularity). The "low"–energy couplings are found to be insensitive to how large the "high"–energy couplings are. These considerations enabled the authors of Refs. [2,4,5] to compute $\sin^2\theta_w(\Lambda_F)$ and $\alpha_3(\Lambda_F)$ (Ref. [5] also applied the same method to the Yukawa sector). A crucial ingredient in the MPP scenario is a relatively large number of standard quark and lepton families. In general, this number is between 8 and 10 in order for $\sin^2\theta_w(\Lambda_F)$ and $\alpha_3(\Lambda_F)$ to have reasonable values. Many of the extra families were given mass of $O(\Lambda_F)$. Another crucial element is the fact that no Grand Unified gauge group was assumed in the computation of $\sin^2\theta_w(\Lambda_F)$.

The simplest extension of the standard model with left–right symmetry is $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)'$. We now apply the MPP scheme to this extended model with, in mind, the calculation of $\sin^2\theta_w(\Lambda_F), \alpha_3(\Lambda_F)$, and the connection of the right–handed breaking scale Λ_R to the Fermi scale Λ_F.

The following set of fermion and scalar fields are chosen following Ref. [1]. For the leptons, we have $\psi_L^\ell = (1,2,1,-1)$ and $\psi_R^\ell = (1,1,2,-1)$ while for the quarks we have $\psi_L^q = (3,2,1,\frac{1}{3})$ and $\psi_R^q = (3,1,2,\frac{1}{3})$. Notice that, here, we identify $U(1)'$ with $U(1)_{B-L}$, where B and L are the baryon and lepton numbers respectively. Since in this case $Q = T_{3L} + T_{3R} + \frac{B-L}{2}$, the last entries in $\psi_{L,R}^{\ell,q}$ denote the $B-L$ quantum numbers. The set $\psi_{L,R}^{\ell,q}$ forms a standard family where one now also has a right–handed neutrino.

The Higgs fields include the following sets:

1. $\phi = (1,2,2,0), \Delta_L = (1,2,1,1), \Delta_R = (1,1,2,1)$ or
2. $\phi = (1,2,2,0), \Delta_L = (1,3,1,2), \Delta_R = (1,1,3,2)$.

The first set $(\phi, \Delta_L, \Delta_R)$ gives only Dirac masses to the neutrinos which are comparable to those of the charged leptons. In this case, to explain the smallness of the neutrino masses, one has to put Majorana mass terms in by hand and use the see–saw mechanism to obtain the mass eigenvalues. This process, however, violates gauge invariance and is unnatural even if we had $M\nu_R^T\nu_R$ with $M \lesssim \Lambda_R$. The second set is more interesting and natural. As pointed out by Ref. [1], this set not only gives Dirac masses to the neutrinos through the coupling with ϕ, but it also gives Majorana masses through the couplings with Δ_L and Δ_R. We will come back to this point later in this paper.

With the above set of Higgs fields whose number is left arbitrary, it can be seen that the pattern of symmetry breaking is as follows: $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow_{\Lambda_R} SU(2)_L \times U(1)_y \rightarrow_{\Lambda_F} U(1)_{em}$. The one–loop renormalization group (RG) equations describing the evolution of the couplings above Λ_R are given by $\frac{d\alpha_{2L,R}}{dt} = \tilde{b}_{2L,R}\alpha_{2L,R}^2, \frac{d\tilde{\alpha}_1}{dt} = \tilde{b}_1\tilde{\alpha}_1^2$ and, for $\Lambda_F \leq E \leq \Lambda_R$, by $\frac{d\alpha_{2L}}{dt} = b_{2L}\alpha_{2L}^2, \frac{d\alpha_y}{dt} = b_y\alpha_y^2$, where $\alpha_i = g_i^2/4\pi$ and $\tilde{\alpha}_1$ and α_y correspond to $U(1)_{B-L}$ and $U(1)_y$ respectively. Also, $t = \ell n(\mu^2/M^2)$. The R.G. coefficients are given as $\tilde{b}_{2L} = b_{2L} = \frac{1}{12\pi}(-22 + \delta(\Delta_L) + n_\phi + 4n), \tilde{b}_{2R} = \frac{1}{12\pi}(-22 + \delta(\Delta_R) + n_\phi + 4n), \tilde{b}_1 = \frac{1}{12\pi}(\frac{8}{3}n + \bar{\delta}(\Delta_L) + \bar{\delta}(\Delta_R)), b_y =$

$\frac{1}{12\pi}(\frac{20n}{3}+\bar{\delta}(\Delta_L)+n_\phi)$, where n is the number of families, n_ϕ the number of $\phi(2,2,0)$ Higgs fields, and, in general, left-right symmetry implies $\overset{(-)}{\delta}(\Delta_L)=\overset{(-)}{\delta}(\Delta_R)=\delta(\Delta)=n_\Delta/2$ when $\Delta_{L,R}$ are doublets and $\delta(\Delta_L)=\delta(\Delta_R)=\delta(\Delta)=2n_\Delta$ and $\bar{\delta}(\Delta_L)=\bar{\delta}(\Delta_R)=\frac{3}{2}\delta(\Delta)=3n_\Delta$ when $\Delta_{L,R}$ are triplets.

Since the theory is non-asymptotically free above Λ_F and since $\alpha_3(\Lambda_F)$ is expected to be of order 0.1, one has to include the two loop contributions to the QCD β-function. Explicitly, one has $\frac{d\alpha_3}{dt}=b_3\alpha_3^2+C_3\alpha_3^3+0(\alpha_3^2\alpha_f,\alpha_3^2\alpha_i(i=1,2))$, where $\alpha_f=g_f^2/4\pi$ with g_f being Yukawa couplings. The coefficients b_3 and C_3 are given by $b_3=\frac{1}{12\pi}(-33+4n), C_3=\frac{1}{48\pi^2}(76n-306)$. In what follows, we will neglect the $\alpha_3^2\alpha_f$ and $\alpha_3^2\alpha_i$ terms. In fact, for the standard model with one Higgs doublet, the results of Ref. [5] show that this is a reasonable approximation as long as one does not have superheavy standard fermions with mass much larger than the $SU(2)_L$ symmetry breaking scale. From here on, we will assume that such is the case.

We now make the MPP assumption that all couplings grow large at a common scale Λ_G. What this statement means in what follows is simply that the term $\alpha_i^{-1}(\Lambda_G)$, which appears in the solution to the one-loop equation, namely $\alpha_i^{-1}(\Lambda_F)=\alpha_i^{-1}(\Lambda_G)+b_i\ell n\frac{\Lambda_G^2}{\Lambda_F^2}$, will be neglected so that $\alpha_i^{-1}(\Lambda_F)\simeq b_i\ell n\frac{\Lambda_G^2}{\Lambda_F^2}$. We can then readily find

$$\sin^2\theta_w(\Lambda_F)=\left(\frac{\alpha_{e.m.}(\Lambda_F)}{6\pi}\right)(-22+\delta(\Delta_L)+n_\phi+4n)\,\ell n\frac{\Lambda_G}{\Lambda_F}. \quad (.1)$$

The value of $\sin^2\theta_w(\Lambda_F)$ is given in Table 1 for various Λ_G, n, n_ϕ and $\delta(\Delta_L)$. We have used $\alpha_{e.m.}(\Lambda_F)=1/128$. Since, at Λ_R, $SU(2)_R\times U(1)_{B-L}$ is assumed to break down to $U(1)_y$ one has $\alpha_y^{-1}(\Lambda_R)=\alpha_{2R}^{-1}(\Lambda_R)+\tilde{\alpha}_1^{-1}(\Lambda_R)$. Upon using $\alpha_y^{-1}(\Lambda_F)=\alpha_y^{-1}(\Lambda_R)+\frac{1}{6\pi}(\frac{20}{3}n+\delta(\Delta_L)+n_\phi)\ell n\frac{\Lambda_R}{\Lambda_F}$, we obtain

$$\frac{\Lambda_R}{\Lambda_G}=exp\left\{\left[\left(-22+\delta+\bar{\delta}+2n_\phi+\frac{32}{3}n\right)\ell n\frac{\Lambda_G}{\Lambda_F}-6\pi\alpha_{e.m.}^{-1}(\Lambda_F)\right]/\left[-22+\delta+\bar{\delta}\right]\right\}, \quad (.2)$$

where left-right symmetry has been used, i.e., $\overset{(-)}{\delta}(\Delta_L)=\overset{(-)}{\delta}(\Delta_R)$. Various values of Λ_R are given in Table 1. Integrating the two-loop equation for α_3, we obtain the values of α_3 which are also shown in Table 1.

The following results emerge. The minimal model with $n_\phi=n_\Delta=1$ can only accommodate $n=8,9$ standard families. For $n\leq 7$, the one-loop term $b_3\alpha_3^2$ dominates over the two-loop term $C_3\alpha_3^3$ for $\alpha_3(\Lambda_F)\simeq 0.1$ and since, in this case,

$b_3 < 0, SU(3)_c$ is asymptotically free above Λ_F contrary to our assumption. As for $n = 10$, the minimal model predicts $0.056 \lesssim \alpha_3(\Lambda_F) \lesssim 0.06$ where the lower bound corresponds to $\sin^2\theta_w^{max}(\Delta_F) \simeq 0.24$ and the upper bound corresponds to $\sin^2\theta_w^{min}(\Lambda_F) \simeq 0.217$ (these values of $\sin^2\theta_w$ are obtained from the measurements of M_w and M_z). The predictions for $\alpha_3(\Lambda_F)$ are too small and one can safely say that, in the minimal model with $\Delta_{L,R}$ being triplets, the number of standard families is 8 or 9. The same conclusion holds for the case when $\Delta_{L,R}$ are doublets (here $0.053 \lesssim \alpha_3(\Lambda_F) \lesssim 0.056$ for $n = 10$). One can lower a little bit the value of $\sin^2\theta_w^{min}(\Lambda_F)$, to say 0.21, with the effect that the upper bound on $\alpha_3(\Lambda_F)$ is altered very little.

Before we discuss the possible values of Λ_R, it is important to see which values of Λ_G are reasonable since Λ_R depends on Λ_G (Eq. (2)). The criteria will be $\sin^2\theta_w(\Lambda_F)$ and $\alpha_3(\Lambda_F)$. We have stated above that $\alpha_3(\Lambda_F) \lesssim 0.06$ is too small. The question is how well does one know $\alpha_3(\Lambda_F)$. Including the two-loop corrections to α_3, one finds $\alpha_3(M^2) = \frac{1}{b_3 t}[1 + \frac{c_3}{b_3^2}\frac{\ell n t}{t}] + 0(t^{-2})$ where $t = \ell n \frac{M^2}{\Lambda_{QCD}^2}$. Although there is no firm handle on the precise value of Λ_{QCD}, the general consensus is for the value of Λ_{QCD} in the $\bar{MS}$ scheme to be around 200 MeV. Measurements of Υ-decay give, on the average, the value $\Lambda_{\bar{MS}}^{(4)} = 200 \pm 50$ MeV[6]. When we "run" α_3, we will use the convention in which $\Lambda_{\bar{MS}}^{(n_f)}$ is flavor-dependent. For various values of $m_t, \alpha_3(\Lambda_F)$ can be computed for three generations with mass $\lesssim$ 250 GeV. The result is $0.09 \lesssim \alpha_3(\Lambda_F) \lesssim 0.1$. However, since the uncertainty in $\Lambda_{\bar{MS}}^{(4)}$ can run from 100 MeV to 300 MeV (approximately) and since there is also a possibility that a fourth generation with mass $\lesssim$ 250 GeV exists, we will consider as a reasonable range for $\alpha_3(\Lambda_F)$ the following one $0.08 \lesssim \alpha_3(\Lambda_F) \lesssim 0.12$.

As for $\sin^2\theta_w(\Lambda_F)$, we shall consider as reasonable the range $0.21 \lesssim \sin^2\theta_w(\Lambda_F) \lesssim 0.24$.

With the above constraints in mind, we now first examine the predictions of the minimal model concerning Λ_R. We require that $\Lambda_F < \Lambda_R < \Lambda_G$. In fact, eq. (2) tells us that, in the minimal model and for a fixed number of families (8 or 9), there is a minimum value of Λ_G for which $\Lambda_R = \Lambda_G$. As Λ_G increases from that minimal value, Λ_R decreases. This is shown in Table 1 where the smallest value of Λ_R corresponds to $\sin^2\theta_w(\Lambda_F) = 0.24$. One can readily see that the best prediction of the minimal model with $\Delta_{L,R}$ being triplets is given by $n = 8, \Lambda_G = M_p = 1.2 \times 10^{19}\ GeV, \Lambda_R \simeq 9 \times 10^{11}$ GeV, $\alpha_3(\Lambda_F) \simeq 0.117$ and $\sin^2\theta_w(\Lambda_F) \simeq 0.207$. From

Table 1, one notices that when $\Delta_{L,R}$ are doublets, one gets either too small a value of $\alpha_3(\Lambda_F) \simeq 0.07$ for $\sin^2\theta_w(\Lambda_F) \simeq 0.216$ or a too small value of $\sin^2\theta_w(\Lambda_F) \simeq 0.182$ for $\alpha_3(\Lambda_F) \simeq 0.117$. In summary, the minimal left–right model with 8 families and one $\phi = (2,2,0), \Delta_L = (3,1,2), \Delta_R = (1,3,2)$ predicts $\Lambda_R \simeq 9 \times 10^{11}$ GeV. As for the W_R mass, left–right symmetry implies $g_L = g_R$ and, at $\Lambda_R, g_L = g_R \simeq 1.11$ giving $M_{w_R} = \frac{1}{2}g_R\Lambda_R \simeq 5 \times 10^{11}$ GeV. We shall talk about M_{w_R} more, when we discuss the neutrino masses. The question of immediate importance is: how small can Λ_R become? It is clear that one has to go beyond the minimal model in order to lower the value of Λ_R.

The simplest extension of the minimal model is the situation in which there are more than one set of Higgs scalars. We have looked at the following cases: 1) $n_\phi = n_\Delta \geq 2$ and 2) $n_\Delta = 1, n_\phi \geq 2$. We first concentrate on the more interesting version of the model in which $\Delta_{L,R}$ are triplets. Again, the requirement $\Lambda_F < \Lambda_R < \Lambda_G$ is imposed. The results are shown in Table 1.

With the constraints $0.21 \lesssim \sin^2\theta_w(\Lambda_F) \lesssim 0.24$ and $0.08 \lesssim \alpha_3(\Lambda_F) \lesssim 0.12$, again only $n = 8 or 9$ gives acceptable results. For case (1) with $n_\phi = n_\Delta \geq 2$, we have the following best results (lowest values of Λ_R): $n = 8, n_\phi = n_\Delta = 2, \Lambda_G = 1.3 \times 10^{18}\ GeV, \Lambda_R = 1.4 \times 10^4\ GeV, \sin^2\theta_w(\Lambda_F) = 0.24, \alpha_3(\Lambda_F) = 0.119; n = 9, n_\phi = n_\Delta = 2, \Lambda_G = \Lambda_R = 2 \times 10^{14}\ GeV, \sin^2\theta_w(\Lambda_F) = 0.227, \alpha_3(\Lambda_F) = 0.08$. For both $n = 8$ and $n = 9, n_\phi = n_\Delta \geq 3$ is unacceptable.

For case (2) with $n_\Delta = 1, n_\phi \geq 2$, the best results (lowest Λ_R) are given by: $n = 8, n_\Delta = 1, n_\phi = 3, \Lambda_G = 10^{19}\ GeV, \Lambda_R = 1.1 \times 10^8\ GeV, \sin^2\theta_w(\Lambda_F) = 0.237, \alpha_3(\Lambda_F) = 0.117; n = 9, n_\Delta = 1, n_\phi = 5, \Lambda_G = \Lambda_R = 1.5 \times 10^{14}\ GeV, \sin^2\theta_w(\Lambda_F) = 0.236, \alpha_3(\Lambda_F) = 0.08$. For $n = 8$ and $9, n_\phi \geq 4$ and $n_\phi \geq 6$ are unacceptable respectively.

The less interesting case of doublet $\Delta_{L,R}$ gives the best value for $n_\phi = n_\Delta = 3$ and $n = 8, \Lambda_G = 10^{19}\ GeV, \Lambda_R = 8.8 \times 10^{10}\ GeV, \sin^2\theta_w(\Lambda_F) = 0.23$, and $\alpha_3(\Lambda_F) = 0.117 (n_\phi = n_\Delta \geq 4$ is unacceptable). For $n = 9$, one has $n_\phi = n_\Delta = 5, \Lambda_G = \Lambda_R = 1.5 \times 10^{14}\ GeV, \sin^2\theta_w(\Lambda_F) = 0.241, \alpha_3(\Lambda_F) = 0.08$ (higher values of $n_\phi = n_\Delta$ are unacceptable).

An examination of the above results and Table 1 reveals that the lowest value of Λ_R one can get is around 14 TeV. This correspond to an interesting lower bound on M_{W_R}, namely ($M_{W_R} = \frac{1}{2} g_{L,R} \Lambda_R$):

$$M_{W_R} \gtrsim 8\, TeV \tag{3}$$

We would like to stress here the fact that the above lower bound on M_{W_R} comes solely, within the MPP framework, from $\sin^2\theta_w$ and α_3. The implication on neutrino masses is discussed next.

Let us reiterate the results obtained earlier. In the minimal model with $n_\phi = n_\Delta = 1$ and $n = 8$ generations, $M_{W_R} \gtrsim 5\times 10^{11}\, GeV$, consistent with $\sin^2\theta_w(\Lambda_F)$ and $\alpha_3(\Lambda_F)$. In the non–minimal case $(n_\phi = n_\Delta = 2, n = 8), M_{W_R} \gtrsim 8\, TeV$. These are the bounds obtained solely from $\sin^2\theta_w(\Lambda_F)$ and $\alpha_3(\Lambda_F)$ regardless of how neutrino masses come about.

In the left–right symmetric model considered here, fermion masses are obtained through the coupling with the Higgs fields ϕ and $\Delta_{L,R}$ (if $\Delta_{L,R}$ are triplets)[1]. These Yukawa couplings are of the form: $\bar\psi_L\phi\psi_R, \bar\psi_L\tilde\phi\psi_R (\tilde\phi = \tau_2\phi^*\tau_2)$ for both quarks and leptons; and, if $\Delta_{L,R}$ are triplets, $\psi_L^T C\tau_2\Delta_L\psi_L, \psi_R^T C\tau_2\Delta_R\psi_R$ for leptons only (C is the Dirac charge–conjugation matrix). Let us look specifically at the Yukawa couplings of a single lepton family. Following Ref. [1] the most general couplings are given by $\mathcal{L}_y = h_1\bar\psi_L\phi\psi_R + h_2\bar\psi_L\tilde\phi\psi_R + ih_3(\psi_L^T C\tau_2\Delta_L\psi_L + \psi_R^T C\tau_2\Delta_R\psi_R) + h.c.$. The minimization of the potential[1] gives $<\Delta_L> = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, <\Delta_R> = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}, <\phi> = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$ where, in general, $\kappa' \ll \kappa$ (suppression of $W_L - W_R$ mixing), $v_R \gg \kappa$, and $v_L = \gamma\kappa^2/v_R \ll \kappa$($\gamma$ is the ratio Higgs self–coupling). The charged lepton mass is given by $m_{\ell^-} = h_1\kappa' + h_2\kappa$ while the mass matrix of the neutral lepton sector, coming from $(\nu^T N^T) MC \begin{pmatrix} \nu \\ N \end{pmatrix} + h.c. (N \equiv C(\bar\nu_R)^T)$, takes the form $M = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$ where $a = h_3 v_L, b = -h_3 v_R, c = \frac{1}{2}(h_1\kappa + h_2\kappa')$. The diagonalization of M with $\kappa' \ll \kappa$ and $b \gg a, c$ gives the following Majorana masses $m_\nu \simeq (h_3\gamma + \frac{1}{4}\frac{h_1^2}{h_3})\frac{\kappa^2}{v_R}, m_N \simeq -h_3 v_R$. In the limit $\gamma \ll \frac{h_1^2}{h_3^2}, h_2 \simeq \frac{1}{2}h_1, h_3 \simeq \frac{1}{2}g$, one obtains the usual "see–saw" mechanism formula (Case I).

$$m_\nu \simeq m_{\ell^-}^2/M_{W_R} \tag{4}$$

Another limit of interest is $\gamma \ll \frac{h_1^2}{h_3^2}, h_1\kappa \simeq h_2\kappa'$ and $h_3 \simeq 1$ giving (Case II).

$$m_\nu \simeq \varepsilon^2 m_{\ell^-}^2/v_R, \tag{5}$$

where $\varepsilon = \kappa'/\kappa \ll 1$.

Let us first examine the consequences of Eqs. (4,5) on the three standard neutrinos. As we have stated earlier, the minimal model predicts $M_{W_R} \gtrsim 5 \times 10^{11}$ GeV or $\Lambda_R (\equiv v_R) \gtrsim 9 \times 10^{11}$ GeV. Using Eq. (4), we obtain

$$m_{\nu_e} \lesssim 5 \times 10^{-10} eV, m_{\nu_\mu} \lesssim 2 \times 10^{-5} eV, m_{\nu_\tau} \lesssim 6 \times 10^{-3} eV \text{ (minimum model)} \quad (.6)$$

for Case I. Since $\varepsilon \ll 1$, Case II gives even much smaller upper bounds than (6). From (6), it is seen that as far as standard neutrinos are concerned (as well as W_R), there are practically no direct experimental consequences in any foreseeable future. Since we are talking about 8 generations, one may also wonder about the masses of the extra neutral leptons. Even if the charged partner weighs 250 GeV ($= \Lambda_F$), one would get $m_\nu \lesssim 125 eV$ for Case I and smaller bound for Case II. For stable neutrinos, the cosmological upper bound is $\sum_i m_{\nu_i} \lesssim 65 eV$. The extra neutrinos will either have to be lighter than 65 eV or more massive than 4.6 GeV if they are stable, otherwise they will have to decay into lighter neutrinos with the constraint $[m(\nu)]^2 \tau(\nu) \leq 2 \times 10^{20} eV^2$ sec[7]. On the other hand, nucleosynthesis favors $n_\nu \leq 4$ where n_ν is the number of light neutrinos. We have seen above that, in the minimal model, most likely $m_\nu < 65 eV$ if we are dealing with Majorana neutrinos. It appears more likely that the extra neutrinos are Dirac particles weighing in the GeV region, in which case a discrete symmetry can be imposed to forbid them to couple to $\Delta_{L,R}$.

In the non–minimal case where there is a chance of observing the right–handed W–boson ($M_{w_R} \gtrsim 8$ TeV), Case I (eq. (4)) predicts

$$m_{\nu_e} \lesssim 0.03 eV, m_{\nu_\mu} \lesssim 1.3 keV, m_{\nu_\tau} \lesssim 400 keV \quad (.7)$$

These bounds, by themselves, are well below the direct experimental bounds of 40 eV, 250 keV and 70 MeV respectively. However, when one combines them with the cosmological bound on the lifetimes within the framework of left–right symmetric models[7], namely $m_{\nu_\mu} \gtrsim$ 400 MeV, $m_{\nu_\tau} \gtrsim$ 900 keV, one runs into conflict with both the direct experimental bounds and our bounds[7]. One then concludes that both m_{ν_μ} and m_{ν_τ} must be less than 65 eV. It then means that $M_{W_R} \gtrsim 5 \times 10^4$ TeV for Case I ($m_{\nu_e} \lesssim 5 \times 10^{-6} eV, m_{\nu_\mu} \lesssim 0.2 eV, m_{\nu_\tau} \lesssim 65 eV$). $M_{W_R}^{min}$ corresponds to $\sin^2 \theta_w(\Lambda_F) \simeq 0.23, \alpha_3(\Lambda_F) \simeq 0.12$. This value of M_{W_R} makes it virtually unobservable.

For Case II (remember that $v_R \gtrsim$ 14TeV), if we take $\varepsilon \simeq 1.7 \times 10^{-2}$, one has

$$m_{\nu_e} \lesssim 5 \times 10^{-6} eV, m_{\nu_\mu} \lesssim 0.2 eV, m_{\nu_\tau} \lesssim 65 eV. \tag{.8}$$

The above values of m_{ν_e} and m_{ν_μ} are too small to be observed. However, now M_{W_R} can be as low as 8 TeV. This feature holds provided $\kappa' \lesssim 1.7 \times 10^{-2}\kappa$. Such a low value for M_{W_R} has definite attractive experimental possibilities.

The above arguments combined with the nucleosynthesis agrument also suggest that the extra neutrinos are Dirac (rather than Majorana) particles, and they can populate the GeV region. They can perhaps provide some of the missing mass of the universe.

In arriving at the above constraints, we have made use of the coupling of leptons to the Higgs fields ϕ, Δ_L, Δ_R. In particular, we have seen that m_ν can be related directly to the charged lepton mass m_{ℓ^-} without referring to the absolute magnitude of κ, κ'. (Eqs. (4,5)). In the minimal model,$\kappa = M_W/g$ (since $V_L \ll \kappa$). In the non-minimal case, $n_\phi = n_\Delta = 2$ and it is possible that the leptons couple to one of the two ϕ's and the quarks couple to the other one. In this case, the results are the ones obtained above. The point here is that it is possibile to have a "low" mass $W_R(\simeq 8TeV)$ while keeping the light Majorana masses small. In fact, as we have analyzed above, m_{ν_e} and m_{ν_μ} in most cases considered here are too small to be of any direct relevance if we insist that $m_{\nu_\tau} \lesssim 65eV$.

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Table I.

n	$n_\phi = n_\Delta$	Λ_G (GeV)	Λ_R (GeV)	$\alpha_3(\Lambda_F)$	$\sin^2\theta_w(\Lambda_F)$
8		1.2×10^{15}	9×10^{11}	0.117	0.207
		1.00×10^{18}	1.3×10^{15}	0.12	0.194
	1				
9		1.00×10^{17}	1.16×10^{5}	0.07	0.237
		2.2×10^{15}	2.2×10^{15}	0.076	0.201
8		1.3×10^{18}	1.4×10^{4}	0.119	0.24
		$8.9\times^{15}$	8.9×10^{15}	0.127	0.207
	2				
9		1.00×10^{15}	8.0×10^{5}	0.077	0.24
		2.0×10^{14}	2.0×10^{14}	0.08	0.227

Table Captions

Table 1: Table showing various predictions for $\Lambda_{G,R}$, $\alpha_3(\Lambda_F)$ and $\sin^2\theta_W(\Lambda_F)(\Lambda_F \simeq 250GeV)$ when $\Delta_{L,R}$ are triplets under $SU(2)_{L,R}$ respectively.